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QUARK-DIQUARK MATTER EQUATION OF STATE AND COMPACT STAR STRUCTURE

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We present the equation of state (EOS) of quark-diquark matter in the quark mass-density-dependent model. The region of the $2-D$ parameter space inside which this quark-diquark matter is stable against diquark disassembling and hadronization is determined. Motivated by observational data suggesting a high compactness of some neutron stars (NS) we present models based on the present EOS and compare the results with those obtained with previous works addressing a quark-diquark composition based on the MIT Bag model. We show that very compact self-bound stars (yet having $\geq 1M_{\odot}$) are allowed by our EOS even if the diquark itself is unbound.

1. Introduction

The spectacular advances of astronomical instrumentation in the last decade has stimulated a great deal of activity on "old" questions of relativistic astrophysics. Among the latter we find the internal structure of compact stars, which is not only interesting *per se*, but also important for a through knowledge of the QCD diagram in the low-T , high μ region.

One of the most simple forms of (indirectly) investigating the nature of high-density matter, more precisely of the EOS is by obtaining accurate determinations of masses and radii. In fact, leaving aside the possibility of differences due to extreme rotation, and adopting General Relativity as a framework, a comparison of the static models generated by integration of the Tolman-Oppenheimer-Volkoff equations with observed data should reveal a great deal of information about the equation of state. Only recently we have achieved sufficient accuracy to attempt some key tests on selected objects, although it should be acknowledged that three decades of compact star astrophysics had produced quite clever arguments to determine masses and radii, even in the cases in which they proved to be wrong.

A paradigmatic example of the above assertions is the celebrated determination of the binary pulsar mass PSR 1913+16, believed to be accurate to the fourth

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decimal place [1]. Other methods based on combinations of spectroscopic and photometric techniques have been recently perfectioned, and have confirmed that at least one X-ray source is significantly above the "canonical" 1.44 $M_\odot$; namely Vela X-1 for which a value of $1.87^{+0.23}_{-0.17}\,M_\odot$ has been obtained [1]. Some works on compact stars have attracted the attention recently because not only the masses have been determined, but also indications of the radii are available, suggesting a very compact structure.

On the other hand, from a particle physics point of view it is fairly well established that at high temperatures and/or high baryon number densities confinement is effectively disabled. The old question of whether the deconfined matter takes place inside "neutron stars" is not yet settled, although a great deal of interest in this issue emerged from recent calculation of superconducting QCD phases (two-flavor superconducting (2SC) and "color flavor locking" (CFL), see [2] for a review). In fact, there is growing evidence that paired states or *diquarks* (e. g. a spin-0, color-antitriplet bound state of two quarks) might occur as a component in the QCD plasma. This is important for understanding hadron structure and high energy particle reactions. Of course, at extremely high densities diquarks are expected to lose their identity and dissolve into quarks. In the intermediate phase diquarks would be expected to be favored.

Diquark correlations were originally thought to arise in part from spin-dependent interactions between two quarks. Later the non-perturbative interactions were claimed to produce a gap substantially greater than in the '80s, possibly as large as $\sim 100\,MeV$. Although there is no consensus about diquark details, it seems certain that a (ud) quark pair experiences some attraction when in total spin-0 and colour-3* states. Many works assume that the diquark correlation is similar to the one experienced by electrons in a superconductor, with diquarks being the Cooper pairs of QCD [4]. However, we shall not be concerned with the details of the binding of the diquark itself in this work. This is because for compact star astrophysics it is perhaps more important to look for the full free energy (per baryon) behavior in bulk rather than the mass of the diquark individually.

2. The equation of state

The quark mass-density-dependent model (QMDD model) is a phenomenological description that attempts to incorporate the confinement and asymptotic freedom of quarks as a function of the baryonic density of matter. According to lattice calculations [23] and string model investigations [24], the quark-quark interaction is proportional to the distance so that $m \propto n_B^{-1/3}$ may be assumed [12]. Therefore, it is reasonable to adopt the following parametrizations of the particle masses. For the quarks u and d we define as usual [8,9,10,11,12,13]

$$m_u = m_d = \frac{C}{n_B^{1/3}}. \tag{1}$$

For diquarks, we introduce here the mass parameter m_{D0} and write the diquark mass m_D as

$$m_D = m_{D0} + m_u + m_d = m_{D0} + \frac{2C}{n_B^{1/3}}. \tag{2}$$

The non-perturbative character of the interactions between quarks and diquarks is included through this particular mass parametrization with the baryon number density.

Before describing the equation of state in detail we would like to make a number of qualitative remarks: To make contact with current calculations we must determine to which extent the model describes the expected diquark physics. The diquark pairing gap that arises from current calculations [2] clearly depends on the chemical potential (or on the baryon number density). Qualitatively the gap is zero at low densities, reaches a maximum value in the 10-100 MeV range at intermediate densities and goes to zero at high densities. To mimic this behaviour the parameter m_{D0} should have the same shape (but with negative sign) as a function of, for example, the baryon number density, even though in QMDD models it is introduced to simulate confinement only. We shall consider only constant values for m_{D0} in this work. As we shall see below this approximation is a reasonable starting point for the physics of interest to us here. On the other hand we must note that recent lattice simulations show that the diquark could be unbound [3] The latter situation can be described by the present model with positive values of m_{D0}, which are also included in the analysis (see below). We must remark that for negative values of m_{D0} the diquark mass becomes negative (as expected) for some baryon number density beyond which the model is no longer valid. However, this n_B is very high for typical values of m_{D0} (for example, m_{D0} = -100 MeV gives $n_{B,lim}$ greater than $12 \times n_0$, with $n_0 = 0.16$ fm^{-3} the nuclear saturation density. We note that our quark-diquark phase may be directly associated to the phase explicitely addressed by Rapp et al. [22] and termed QDQ, as a state intermediate between hadrons and 2SC, but an identification with the 2SC may also be possible when the masses go to zero at high density. Although strictly speaking this QDQ and 2SC with zero net strangeness would be ustable against weak interaction decays in neutron stars, the number of strange quarks that is produced in the mixture can be neglected in a first approximation. The finite mass $m_s \sim 150$ MeV suppress the ocupation of the corresponding Fermi sea, while the occupation of the bosonic QDQ or 2SC ground state is strongly favoured energetically. We must note that a similar mass parametrization ($m_s = m_{s0} + C/n_B^{1/3}$) has been employed to describe strange quark matter within the QMDD model. Also note the slightly different parametrization than the one employed in Ref. [15] motivated by an intuitive construction of the mass formula.

In view of the above, we shall proceed to model the quark - diquark plasma as a gas of u and d quarks, and (ud) diquarks. Non-perturbative effects are accounted

for using the quark mass-density-dependent model of confinement to derive the properties of the equation of state.

The expression for the thermodynamic potential density of particles of mass m_i, chemical potential μ_i and degeneracy factor g_i in the large volume limit of a free gas is

$$\Omega_i = \frac{g_i T}{2\pi^2} \int_0^\infty \ln(1 \pm e^{(\mu_i - (p^2+m_i^2)^{1/2})/T)}) p^2 dp \tag{3}$$

Because of the dependence of the mass with density n_B in the QMDD model the standard relation $P = -\Omega$ is no longer valid. [12,13,8,9] The pressure is actually given by

$$P = -\frac{\partial(V\Omega)}{\partial V} = \sum_i \left(-\Omega_i + n_B \frac{\partial m_i}{\partial n_B} \frac{\partial \Omega_i}{\partial m_i} \right). \tag{4}$$

For the other thermodynamical quantities we write (see Ref.[12])

$$E = \Omega + \sum_i \mu_i n_i - T\frac{\partial \Omega}{\partial T} \tag{5}$$

$$n_i = -\frac{\partial \Omega}{\partial \mu_i}\Big|_{T,m_k} \tag{6}$$

with E the energy density and n_i the particle number density of the i-species.

As it has been stressed before [8,9], for the fermionic component the term $n_B \frac{\partial m_i}{\partial n_B} \frac{\partial \Omega_i}{\partial m_i}$ forces the pressure to be zero at finite baryon number, playing the role of the bag constant B of the MIT Bag Model. Needless to say, this is a property related to the mass parametrization only, and has no relation with the statistics of the particles themselves (Bose or Fermi). However, it happens that in the case of bosons this extra term does not contribute at zero temperature. Thus, as in the standard case, for massive bosons at $T = 0$ the energy density reduces to $E = \sum_i n_i m_i$.

As we mentioned above we shall model the quark - diquark plasma at zero temperature as a mixture of a Fermi relativistic gas of u and d quarks, and a Bose condensate of (ud) diquarks. We neglect the contribution of antiparticles and that of a leptonic component such as electrons, muons or neutrinos. This mixture is then described by [16,17]

$$P = \sum_{i=u,d} m_i n_i x_i^2 G(x_i) - \sum_{i=u,d} m_i n_i f(x_i) \tag{7}$$

Diquarks do not contribute to the pressure in this approximation, since they are all in the ground state, but do contribute to the energy density

$$E = \sum_{i=u,d} m_i n_i F(x_i) + m_D n_D. \tag{8}$$

The number density of quarks u and d is given by

$$n_i = \frac{1}{\pi^2} m_i^3 x_i^3. \tag{9}$$

where the functions F, G and f are given by [8,13]

$$F(x_i) = \frac{3}{8}\left[x_i(x_i^2+1)^{1/2}(2x_i^2+1) - \arg\sinh(x_i)\right]/x_i^3 \tag{10}$$

$$G(x_i) = \frac{1}{8}\left[x_i(x_i^2+1)^{1/2}(2x_i^2-3) + 3\arg\sinh(x_i)\right]/x_i^5 \tag{11}$$

$$f(x_i) = -\frac{3}{2}\frac{n_B}{m_i}\frac{dm_i}{dn_B}\left[x_i(x_i^2+1)^{1/2} - \arg\sinh(x_i)\right]/x_i^3 \tag{12}$$

being $x = p_F/m_i = (\mu_i^2 - m_i^2)^{1/2}/m_i$, μ_i the chemical potential and m_i the mass of the particle, which in our case depends on the baryon density.

These equations must be complemented with the following conditions:

1) Chemical equilibrium: $D \rightleftharpoons u + d$ requires $\mu_D = \mu_u + \mu_d$. If we use the fact that at $T = 0$ the condensed diquarks all have zero kinetic energy we have ($\mu_D = m_D$)

$$m_D = \mu_u + \mu_d. \tag{13}$$

Then, using the fact that, $\mu = (x^2+1)^{1/2}m$, $m_u = m_d$, and $m_D = m_{D0} + 2m_u$, we can write the chemical equilibrium as

$$\frac{m_{D0}}{m_u} + 2 = (x_u^2+1)^{1/2} + (x_d^2+1)^{1/2}. \tag{14}$$

2) Electrical charge neutrality implies $1/3n_D + 2/3n_u - 1/3n_d = 0$, therefore, the diquark number density is given by

$$n_D = n_d - 2n_u = \frac{1}{\pi^2} m_u^3 (x_d^3 - 2x_u^3). \tag{15}$$

3) Baryon number is given by $n_B = 2/3n_D + 1/3(n_u + n_d)$, thus

$$n_B = n_d - n_u = \frac{1}{\pi^2} m_u^3 (x_d^3 - x_u^3). \tag{16}$$

The equation of state can be solved numerically for each n_B once the parameters (C, m_{D0}) are given (see Figs 1 and 2).

As the baryon density increases the EOS tends to a pure "quark d-diquark" composition since the number density of quarks u goes to zero and the abundances of quarks d and diquarks D become equal. The same behaviour arises for smaller enough values of the mass m_{D0} (see below). In these cases the EOS takes a very simple form: $x_u = 0$, $n_u = 0$, $n_B = n_D = n_d = (m_u^3 x_d^3)/\pi^2$.

As expected from simple physical considerations, we find that the EOS of the quark-diquark is quite soft, especially at relatively high densities, because of diquark

condensation. The EOS becomes somewhat stiffer at low densities when compared to the MIT-inspired EOS developed in Ref.[18], at least for a large region of the parameter space. The same effect is found if we compare the MIT and the QMDD model EOS for strange quark matter [8,9].

3. Stability of the quark - diquark matter

In order to determine the allowed values for C and m_{D0} we impose the following stability criteria for quark-diquark matter.

1) Instability of two flavor quark matter: The energy per baryon of two flavor quark matter (at $P = 0$ and $T = 0$) must be higher than 939 MeV (the neutron mass). Assuming that the current mass of quarks u and d is zero this results in the condition $C > (159.3\, MeV)^2$.

The energy per baryon of pure udd matter at $P = 0$ and $T = 0$ is, for a given C

$$\left.\frac{E}{n_B}\right|_{udd} = m_u^{'}[F(x_{u0}) + 2F(x_{d0})] \tag{17}$$

where $m_u^{'}$ is the mass of quark u in two flavor quark matter at zero P and T,

$$m_u^{'} = \frac{C}{n_B^{1/3}} = \frac{C}{n_u^{1/3}} = \frac{C^{1/2}\pi^{1/3}}{x_{u0}^{1/2}} \tag{18}$$

and $x_{d0} = 2^{1/3}x_{u0}$, where $x_{u0} = 1.13173096$ is the value that produces a zero-point pressure, $P_{udd} = 0$. This criterion has already been used in previous papers [8,9,12,13].

2) Relative stability of quark-diquark matter: The energy per baryon of quark - diquark matter (hereafter ud-D matter) must be lower than that of pure two flavor quark matter (at $P = 0$, $T = 0$).

The zero pressure condition for quark - diquark matter at $T = 0$ reads

$$x_u^5 G(x_u) - x_u^3 f(x_u) + x_d^5 G(x_d) - x_d^3 f(x_d) = 0. \tag{19}$$

The energy per baryon is given by

$$\left.\frac{E}{n_B}\right|_{ud-D} = m_u \frac{x_u^3 F(x_u) + x_d^3 F(x_d) + (\frac{m_{D0}}{m_u} + 2)(x_d^3 - 2x_u^3)}{x_d^3 - x_u^3} \tag{20}$$

Note that the mass m_u of quark u in $ud - D$ matter is in fact a function of C, x_u and x_d given by $m_u = C/n_B^{1/3} = C/(n_d - n_u)^{1/3}$, from which we derive

$$m_u = \frac{C^{1/2}\pi^{1/3}}{(x_d^3 - x_u^3)^{1/6}}. \tag{21}$$

Also, the factor $(m_{D0}/m_u + 2)$ in Eq. (20) is a function of x_u and x_d given by Eq. (14).

The fundamental relation for this stability requirement is

$$\left.\frac{E}{n_B}\right|_{ud-D} < \left.\frac{E}{n_B}\right|_{udd} \tag{22}$$

therefore, from eqs. (20) and (17) we obtain

$$\frac{x_u^3 F(x_u) + x_d^3 F(x_d) + (\frac{m_{D0}}{m_u} + 2)(x_d^3 - 2x_u^3)}{(x_d^3 - x_u^3)^{7/6}} < \kappa_0 \tag{23}$$

where

$$\kappa_0 = \frac{[F(x_{u0}) + 2F(x_{d0})]}{x_{u0}^{1/2}}. \tag{24}$$

From eqs. (23) and (19) we find numerically the set of values x_{u1} and x_{d1} that satisfy the imposed stability conditions.

Using chemical equilibrium alone (eq. (14)) we find a very simple relation between m_{D0} and C

$$m_{D0} = KC^{1/2} \tag{25}$$

with

$$K = \pi^{1/3} \frac{(x_{u1}^2 + 1)^{1/2} + (x_{d1}^2 + 1)^{1/2} - 2}{(x_{d1}^3 - x_{u1}^3)^{1/6}}. \tag{26}$$

This is an "isochemical" straight line along which the energy per baryon increases with C. The condition $(E/n_B)|_{ud-D} < (E/n_B)|_{udd}$ is fullfilled for $K < 1.723$ (the upper limit is given by the full straight line in Figure 3). Also, we find that for $K < 0.8417$ there are no quarks u in the mixture and the composition is always a pure "quark d-diquark" mixture (see dashed line in Figure 3).

3) Absolute stability condition: we have also looked for a region inside which quark - diquark matter could have even a lower energy per baryon than the neutron mass

$$\left.\frac{E}{n_B}\right|_{ud-D} < 939 MeV, \tag{27}$$

being though absolutely stable. In the $(C^{1/2}, m_{D0})$ parameter space this region is "triangle-like". The same shape has been obtained for strange quark matter in the QMDD model [9]. On the right side boundary of this "triangle-like" region the energy per baryon of quark-diquark matter at zero pressure is exactly 939 MeV. The region is shown in Fig. 3 and labeled with A and A'. It is interesting to note that negative values of m_{D0} are allowed (see Fig. 3) and so the diquark may be bound in this model, that is, its mass will be smaller than the sum of the masses of its constituent particles.

4. Quark-diquark stars

The exploration of the mass-radius relation for compact stars is in fact a potentially powerful tool in testing the existence of different phases of matter inside NSs, and their composition. On the other hand the observational data of the mass-radius relation imposes severe constaints on high density equations of state.

Almost all of the meassured masses of "neutron stars" lie within a narrow range around 1.4 $M_{\odot}$. This mass scale may be due to the fact that neutron stars are formed in the gravitational collapse of supernovae and come just from the iron cores of core collapse supernovae. However, we may wonder whether smaller neutron stars could exist and whether there exist some mechanism capable of create them because these are not expected in core collapse models and hold the potential for discriminating among possible equations of state. This question becomes more significant in view of the recent claim of very low-mass/low-radii objects. At least two objects, Her X-1 and RX J1856-37 are candidates to high compactness. They have been claimed to have radii $\sim 7km$ and a masses around $1M_{\odot}$ [21,14].

If quark- diquark matter has a lower energy per baryon than normal nuclear matter we may expect the existence of stable stars made up entirely by this phase. By solving the Tolman-Oppenheimer-Volkoff equations of stellar structure, we explored the mass- radius relation for quark-diquark stars assuming a parameter set inside the window of absolute stability.

In Figs. 4 and 5 we see that for particular values of the parameters $(C^{1/2}, m_{D0})$ the obtained values of M and R are compatible with present determinations of the above sources. We also find that the effect of increasing m_{D0} for a fixed C is to lower both the maximum radius and the maximum mass of the stellar configurations. Similarly, increasing C at fixed m_{D0} allows configurations with smaller maximum radius and mass. The $(C^{1/2}, m_{D0})$ values selected for the sequences shown in Figs. 4 and 5 match naturally the observed compactness of Her X-1 and RX J1856-37.

5. Conclusions

We derived an equation of state for quark diquark-matter in a self-consistent phenomenological model where the mass of quarks and diquarks is parametrized with the baryon number density. The confining forces are included via a dependence of the mass with $n_B^{-1/3}$. The model depends on two free parameters $(C^{1/2}, m_{D0})$ whose values can be limited invoking stability criteria and allows the existence of a stable state of the quark-diquark gas at zero pressure, for densities of the order of few times the nuclear saturation density (see Figs 1 and 2). The free parameter m_{D0} is introduced in order to describe in a simple form the unknown binding of the diquark. Negative values of m_{D0} should mimic the behaviour of bound color superconducting quark matter while positive values are considered in view of recent results [3] claiming a diquark mass of $\sim$ 700 MeV which suggest that this particle is unbound. The mixture contains a Bose condensate of (ud) diquarks so that, the resulting equation of state is in fact very soft, as it can be seen in Figure 1 where we have depicted P versus E for different parameters . The here described quark-

diquark phase has always a non-zero fermionic pressure, since due to the excess of quarks d over quarks u there will always exist a free Fermi gas of quarks d to satisfy the equilibrium conditions. When compared with the equation of state for quark-diquark matter presented in [18] we found that independently of the value of the parameters, the latter is considerably stiffer at low densities. However, in some cases (c,d, e and f of Fig. 1) there is a limiting density beyond which the new equations of state are softer than the previous one. This is relevant for compact star structure because it allows the existence of stable compact stars with even smaller radius than those calculated in previous models [18]. It has been argued that the few percent effects ($\mathcal{O}\ (\Delta/\mu)^2$) from the condensation are impossible to disentangle from other uncertainties, but the fact is that, for example, the whole strange matter theory is also subject to this caveat, and in this respect the models here presented open a qualitatively similar scenario for compact stars.

We have found the regions in the parameter space $(C^{1/2}, m_{D0})$ where quark-diquark matter in bulk is stable against decaying to pure two flavor quark matter and to a neutron gas (see Fig. 3). The region of absolute stability allows the existence of pure quark-diquark stars. It is important to note that the existence of self-bound compact stars is possible for both bound and *unbound* diquarks, since the assumed bosonic character of the diquark favors the transition of free quarks at the Fermi surface to the bosonic ground state and ensures a global energy decrease, more specifically a lower value of the energy per baryon, for the appropiate choice of parameters. That is, since typical Fermi energies in a two flavor quark mixture is ~ 300 MeV, the energy gained by condensing bosons and sending them to the ground state is in fact more relevant for stellar structure than the hypothetically gained energy of the pair gap. Moreover, even in the case of unbound diquarks a lower energy state is reached by the system as a whole. Therefore, although the parameter that mimics the pair gap may be not only negative but also zero or positive, energy is gained in all cases due to the assumed bosonic character of the superconducting pair. A slightly different parametrization used in [15] produced different results. In particular the resulting stellar models were not so compact, a feature due to a larger stifness of that equation of state for all the parameter range. This gauges the sensitivity of the models to the assumptions on the basic physics.

Finally, we must note that, strictly speaking, quark-diquark matter is unstable against weak interactions producing strangeness. In this sense, the present study may be considered as a simplification analogous to pure neutron matter models of neutron stars. A more general study should include the possibility of strangeness production, the appeareance of leptons in the mixture, as well as 2SC and color flavor locking (CFL). We can expect that the opening of the weak channel will produce an even softer equation of state and so higher compactness of these stars. These improvements will be presented in future work.

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References

1. M.H. van Kerkwijk, in *Proceedings of the ESO Workshop on Black Holes in Binaries and Galactic Nuclei*, eds. L. Kaper, E.M. van den Heuvel and P.A. Woudt (Springer-Verlag, 2001).
2. M. Alford, invited talk to the *Proceedings of the Workshop on Compact Stars and the QCD Phase Diagram*, Copenhagen 2001. astro-ph/0110150, to be published.
3. M. Hess, F. Karsch, E.Learmann and I. Wetzorke, *Phys.Rev.* D58 (1998) 111502
4. S. Fredriksson, M. Jändel and T. I. Larson, *Phys. Rev. Lett.* 81, (1983) 2179.
5. Dey M., Bombaci I., Dey J., Ray S. and Samanta B.C. , *Phys. Lett.* B438, (1998) 123; Addendum (1999), B447, 352; Erratum B467, (1999) 303.
6. Li X., Bombaci I., Dey M., Dey J. & van den Heuvel E. P. J., *Phys. Rev. Lett.* 83, (1999) 3776.
7. Li X., Ray S., Dey J., Dey M. & Bombaci I., *Astrophys. J.* 527, (1999) L51.
8. O. G. Benvenuto and G. Lugones, *Phys. Rev* D 51, (1995) 1989
9. G. Lugones and O. G. Benvenuto, *Phys. Rev* D 52, (1995) 1276
10. G. X. Peng, P. Z. Ning and H. Q. Chiang, *Phys. Rev.* C 56, (1997) 491
11. G. X. Peng, H. C. Chiang, P. Z. Ning and B. S. Zou, *Phys. Rev.* C 59, (1999) 3452
12. G. X. Peng, H. C. Chiang, J. J. Yang, L. Li and B. Liu, *Phys. Rev.* C 61, (2000) 015201
13. G. X. Peng, H. C. Chiang and P. Z. Ning, *Phys. Rev.* C 62, (2000) 02580
14. J. Pons, F. M. Walter, J. M. Lattimer, M. Prakash, R. Neuhäuser, and Penghui An., *Astrophys. J.*, 564, (2002) 981
15. J.E. Horvath, G. Lugones and J.A. de Freitas Pacheco, contributed paper to the *Proceedings of the Workshop on Compact Stars in the QCD Diagram*, Copenhagen 2001,astro-ph/0112159, to be published.
16. S. L. Shapiro and S. A. Teukolsky, *Black holes, white dwarfs, and neutron stars: the physics of compact objects*, (J. Wiley, 1983).
17. S. Chandrasekhar, *An Introduction to the Study of Stellar Structure* (Chicago University Press, Chicago, 1939)
18. J. E. Horvath and J. A. de Freitas Pacheco, *Int. J. of Mod. Phys.* D 7, (1998) 19
19. J. E. Horvath, J. A. de Freitas Pacheco and J. C. N. de Araújo, *Phys. Rev.* D 46, (1992) 4754
20. T. Schafer, *Nucl. Phys.* A642, (1998) 45c
21. X. -D. Li, Z. -G. Lai and Z. -R. Wang, *Astron. Astrophys.* 303, (1995) L1
22. R. Rapp, T. Schafer, E.V. Shuryak and M. Velkovsky, *Ann. Phys.* 280, (2000) 35
23. V. M. Belyaev and Ya. I. Kogan, *Phys. Lett.* B 136, (1984) 273; K. D. Born, E. Laermann, N. Pirch, T. F. Walsh and P. M. Zerwas, *Phys. Rev.* D 40, (1989) 1653
24. N. Isgur and J. Paton, *Phys. Lett.* B 124, 273 (1983); *Phys. Rev.* D 31, (1985) 2910

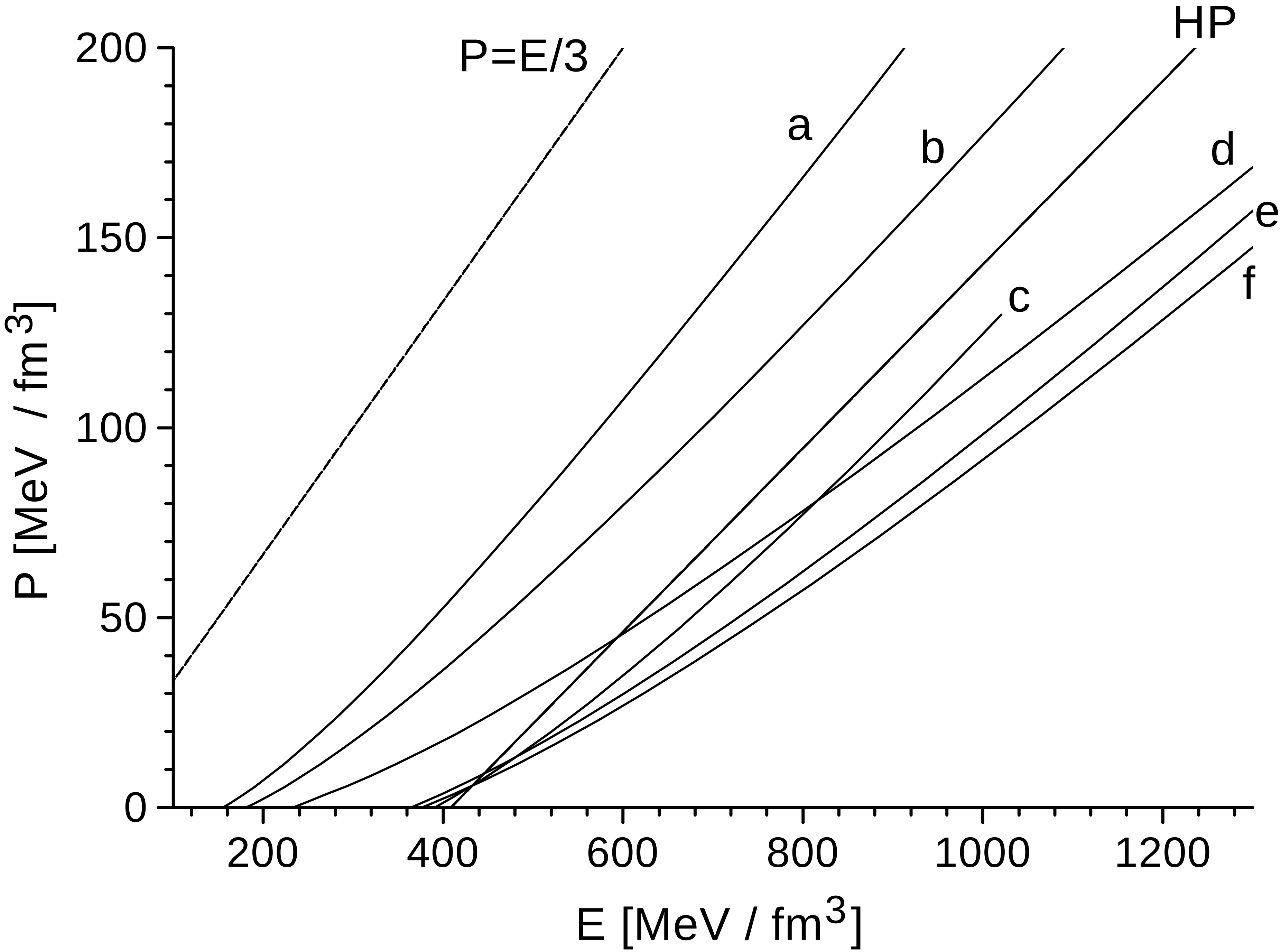
Fig. 1. Equation of state of quark diquark matter in the quark mass density dependent model for different set of parameters $(C^{1/2}, m_{D0})$. The curves are labeled with: a = $(160, -100)$, b = $(160, 0)$, c = $(200, -100)$, d = $(160, 210)$, e = $(185, 100)$ and f = $(185, 130)$. For the sake of comparison we also include the ultrarelativistic EOS $P = E/3$ and the EOS presented in Ref.[18] (labeled as HP).

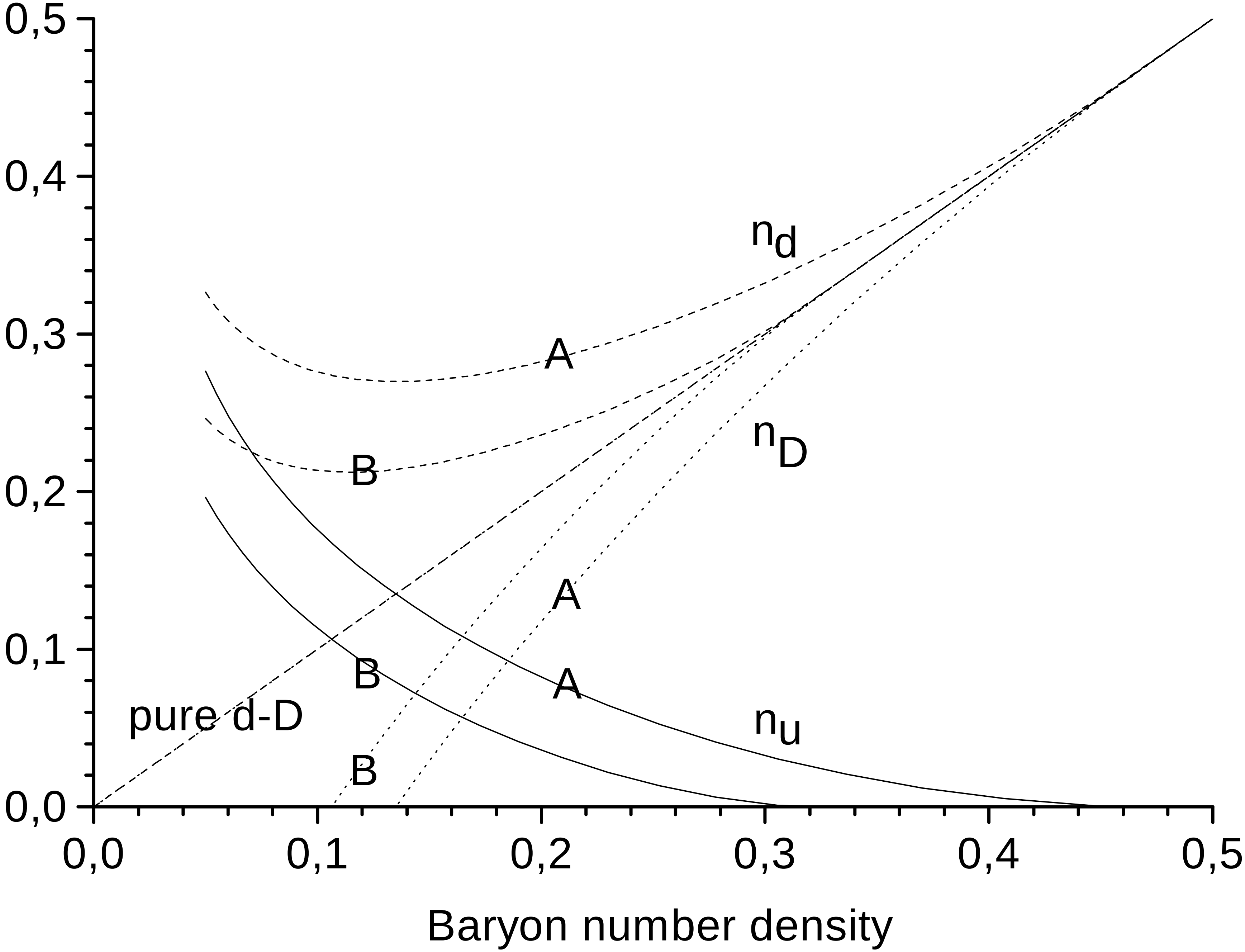
Fig. 2. The particle number densities of u and d quarks and diquarks for the parameters $(160, 200)$ (label A) and $(185, 130)$ (label B) . At high baryon number densities the number of quarks u goes to zero and the number of quarks d and diquarks are equal. We also include the case of pure quark d - diquark matter $n_B = n_d = n_D$ corresponding to any set of parameters in the region A' of Figure 3.

Fig. 3. Different stability regions in the parameter space of the model. Inside regions A and $A^{'}$ quark-diquark matter has a lower energy per baryon than a neutron being so absolutely stable. For parameters inside the region $A^{'}$ there are no u quarks in the mixture, and matter is always composed by quarks d and diquarks. Inside the region B below the full straight line the mixture has a lower energy than pure two flavor quark matter composed only by quarks u and d.

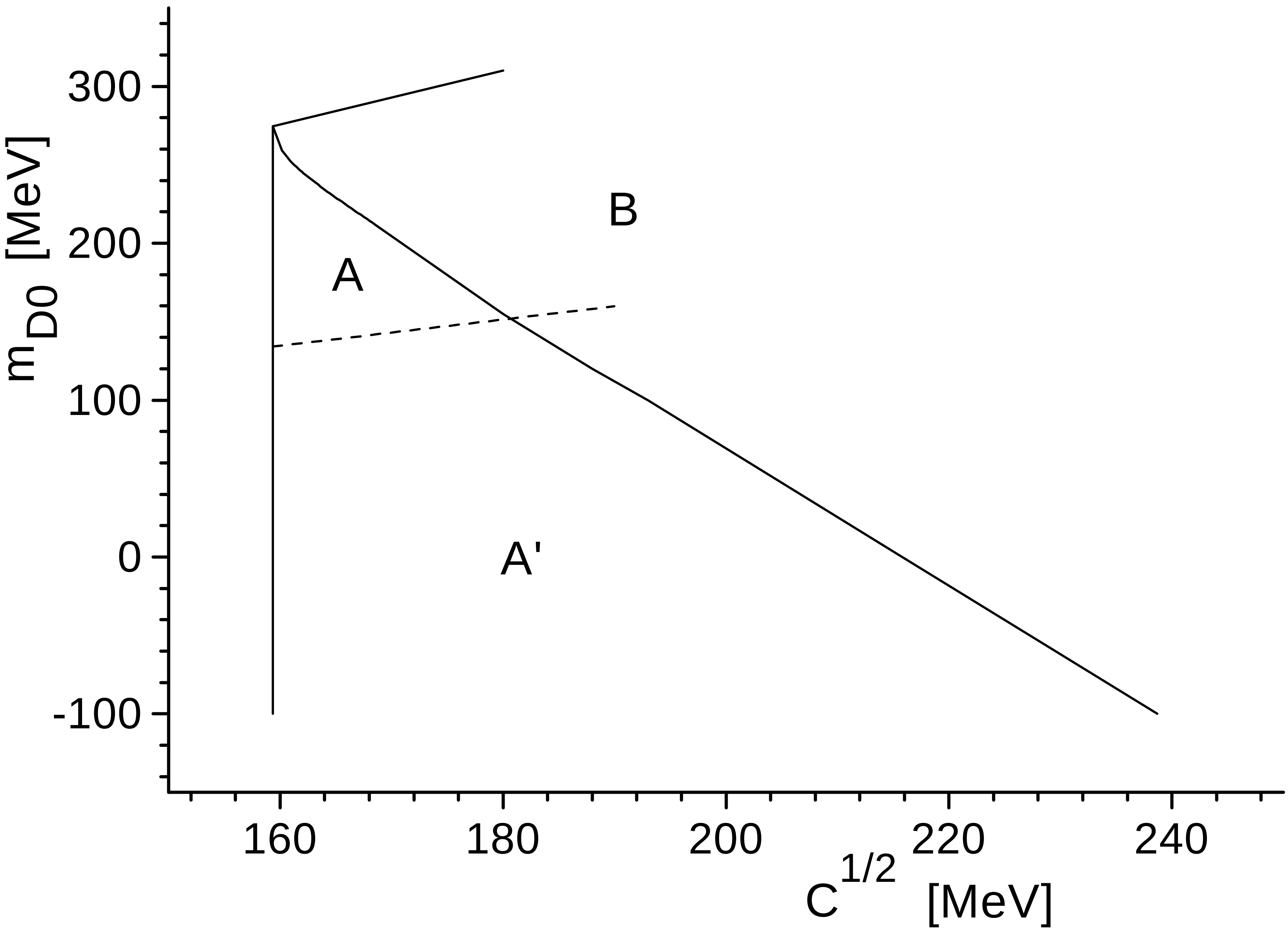
Fig. 4. The mass radius relation for quark-diquark stars. The curve labeled with HP corresponds to the EOS developed in [18]. The other curves correspond to the following set of the parameters $(C^{1/2}, m_{D0})$: a = $(185, 130)$, b=$(185, 100)$, c=$(180, 100)$, d=$(160, 210)$. Models overlap the range of the M-R measurements of Her X-1 and RX J1856-37.

Fig. 5. The same as the previous figure but for A= $(210, -100)$, B= $(200, -100)$, C= $(160, 0)$, D=$(160, -100)$.

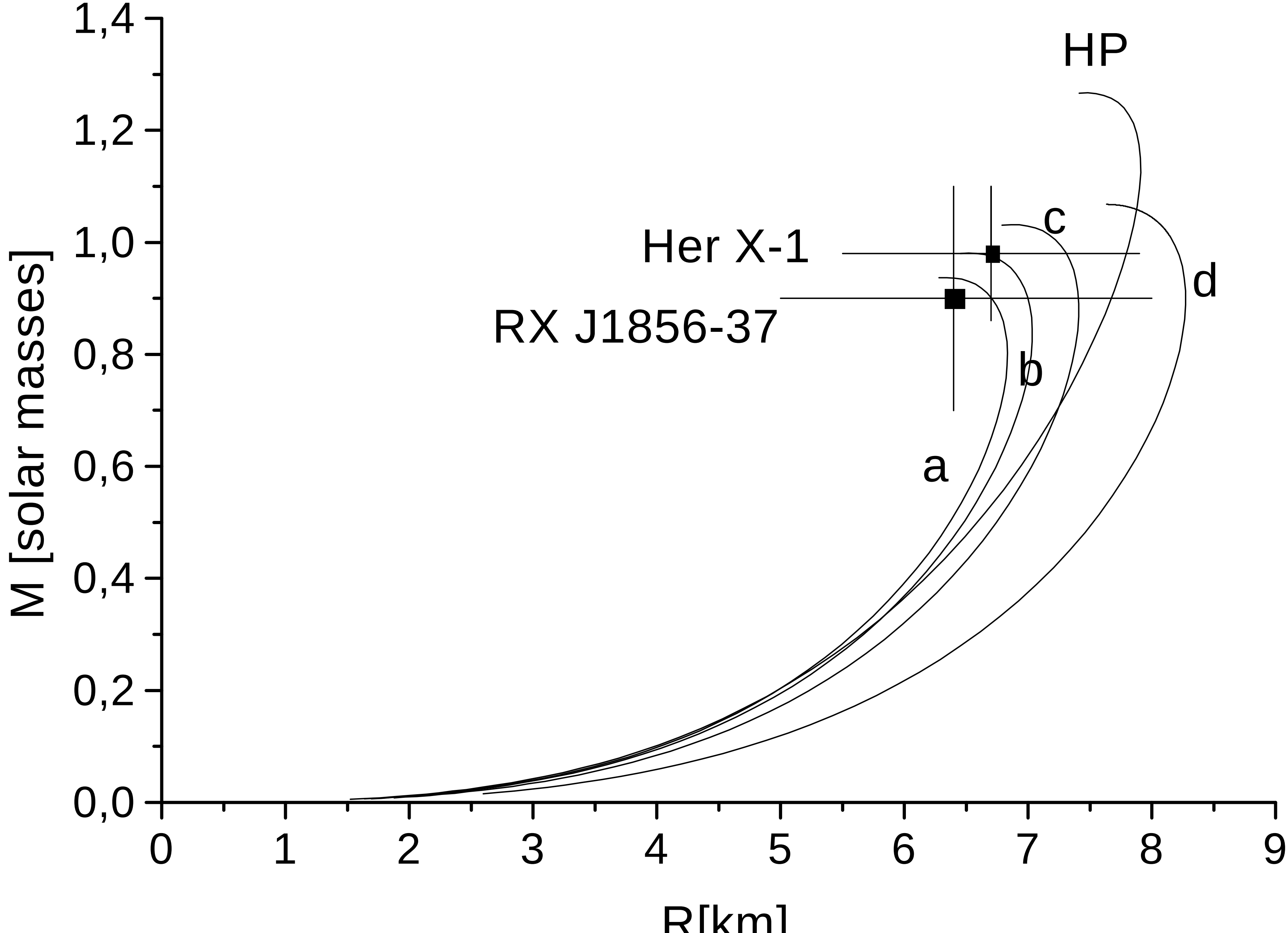

P=E/3
a
b
HP
c
d
e
f
0
50
100
150
200
200
400
600
800
1000
1200
P [MeV / fm^3]
E [MeV / fm^3]

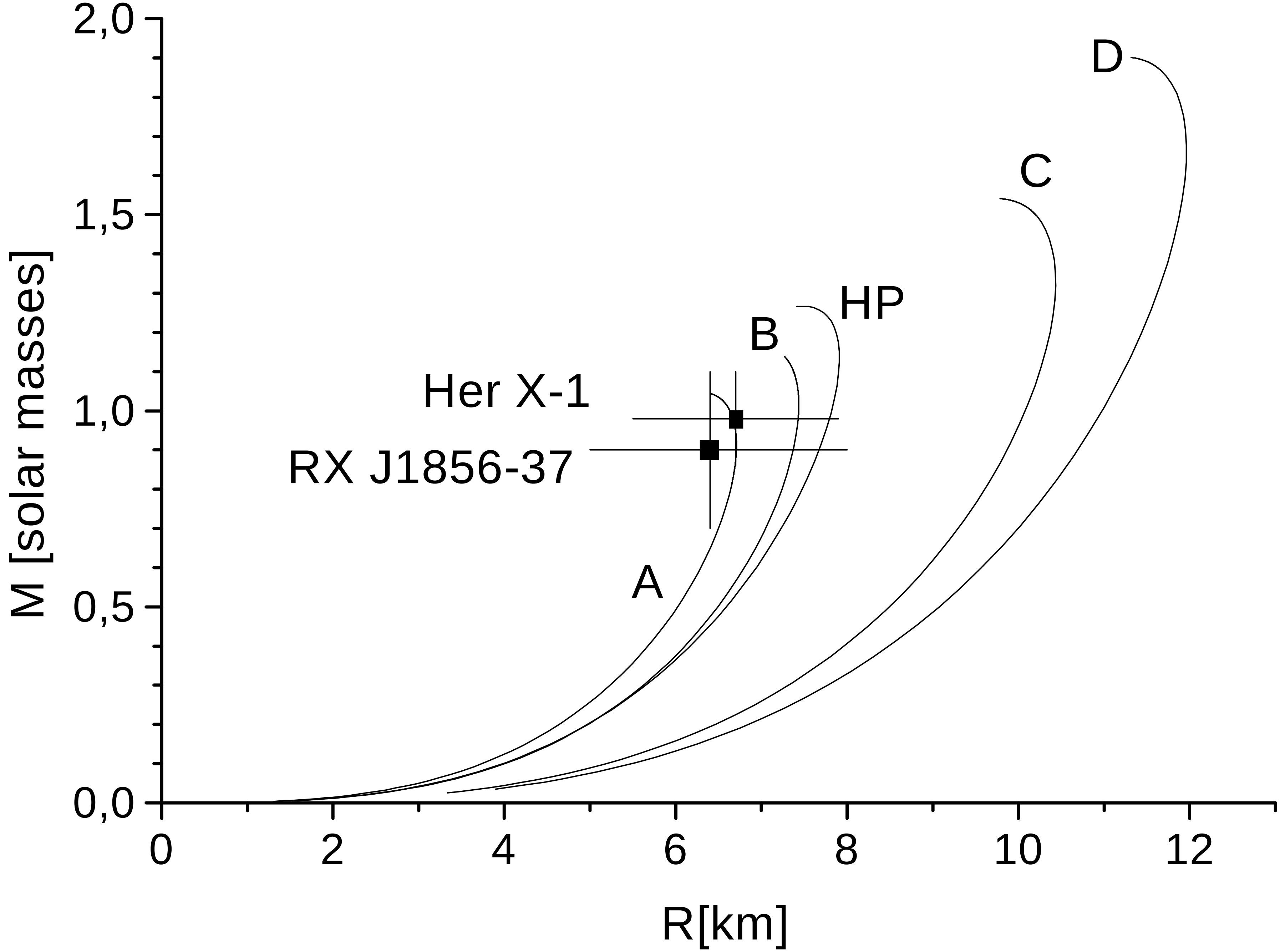


0,5
0,4
0,3
0,2
0,1
0,0
0,0
0,1
0,2
0,3
0,4
0,5
Baryon number density
n_d
A
B
n_D
A
B
A
B
n_u
pure d-D



300
200
100
0
-100
m_{D0} [MeV]
160
180
200
220
240
$C^{1/2}$ [MeV]
A
B
A'


HP
Her X-1
RX J1856-37
a
b
c
d
M [solar masses]
R[km]
0,0
0,2
0,4
0,6
0,8
1,0
1,2
1,4
0
1
2
3
4
5
6
7
8
9



M [solar masses]
R[km]
2,0
1,5
1,0
0,5
0,0
0
2
4
6
8
10
12
D
C
HP
B
Her X-1
RX J1856-37
A